



**Franco-Moroccan
Mathematics Days**

Book of Abstracts

**Franco-Moroccan Mathematics Days (F2MDS'23)
on the occasion of the
International Mathematics Day
under the theme**

Mathematics for everyone and everywhere

**Faculty of Sciences - Tetouan, Abdelmalek Essaadi University
March 13-16, 2023**



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Synopsis

Greater global awareness of mathematical sciences and a strengthening of their teaching, as well as popularization, are vital to address many challenges of tomorrow and to improve the quality of life in both developed and developing countries. That is why, in 2019, UNESCO made the 14th March of each year the "International Day of Mathematics", also known as the "pi-day", π being one of the world's most widely-known mathematical constants, whose value is close to 3.14.

In this context, the Abdelmalek Essaadi University and the Faculty of Sciences of Tetouan, in cooperation with the Sorbonne Paris Nord University and the International Association ALPaGe, organize the first edition of the French-Moroccan Mathematics Days. During these days will be held a variety of scientific activities involving training, workshops, and international conferences on scientific mediation, applied mathematics, innovation and didactics of mathematics. The last day will be a cultural wealth of north Morocco discovery.

The event will be an opportunity to build bridges between mathematical communities, reinforce existing international cooperation's and initiate new ones, hence contributing to the outreach of all involved parties.

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Part I

Keynote Speakers

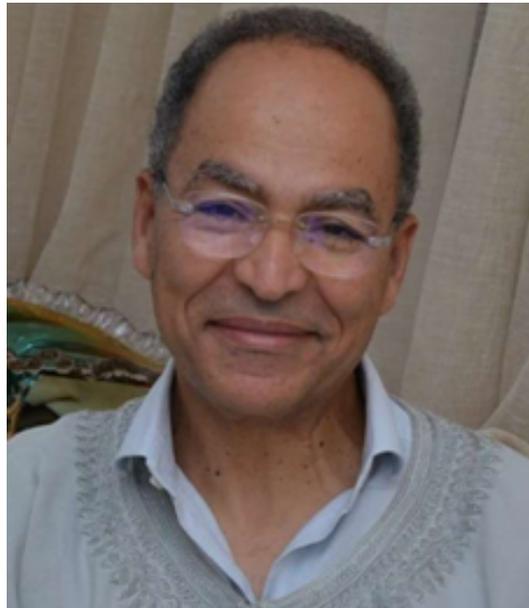
1 émergence de quelques Branches des Mathématiques : Systèmes Dynamiques et Théorie du Chaos

Aziz Alaoui, ¹

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Abstract

Après une excursion dans un passé plus ou moins lointain, relatant chronologiquement, mais brièvement l'apparition des premières formes de mathématiques, un inventaire (un peu à la Prévert) de certaines branches les plus établies en mathématiques sera proposé en s'appuyant parallèlement sur certains noms célèbres auxquels sont attribués les fondements de ces dernières. L'occasion de citer Al Kashi et son approche du nombre π , développée dans " Rissalat-al-muhitiyya (Le traité du cercle). Nous insisterons en particulier sur l'une de ces branches et sur son émergence, celle de la théorie du Chaos, dans le cadre des systèmes dynamiques. Cette dernière relativement active se développe à la frontière de la topologie, de l'analyse, de la géométrie, de la théorie de la mesure et des probabilités. Avant de conclure, s'il faut encore le démontrer sur l'utilité de cette discipline et le rôle incontournable qu'elle joue dans beaucoup de domaines et sur certains métiers auxquels elle conduit



2 Periodic solutions in infinite dimensional dynamical systems: Past and Present

Khalil Ezzinbi¹

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Abstract

The aim of this work is to present a new method to prove the existence of periodic solutions for dynamical systems well-posed in infinite dimensional spaces. We use the Poincaré map to establish the existence of periodic solutions without compactness of the semigroup generated by the linear part, for that goal we use a new fixed point Theorem that we established recently for affine maps. Several applications are given for evolution equations and partial functional differential equations.



3 Endofinitude pour les modules et les espaces de Banach

El Amin Kaidi¹

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Abstract

En partant de résultats élémentaires, bien connus, caractérisant la finitude du cardinal d'un ensemble (resp. De la dimension d'un espace vectoriel) en termes de ses endomorphismes, on a introduit de notions parallèles pour les objets d'une catégorie concrète.

Dans cet exposé on va survoler plusieurs questions liées á ce sujet pour les cas des catégories de modules et d'espaces de Banach. Plusieurs questions ouvertes seront présentées et motivées.



4 Comprendre la fusion nucléaire dans les Tokamaks pour une énergie illimitée dans le futur: les points chauds dans les plasmas expliqués par les Mathématiques des équations différentielles ordinaires

Olivier Lafitte¹

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Abstract

Un des grands espoirs futurs pour la production d'énergie est la fusion de l'hydrogène, non pour délivrer de très grandes quantités d'énergie comme pour une bombe H, mais pour entretenir une réaction de fusion nucléaire comme dans les soleils. Cela passe par le transfert d'énergie entre un champ électromagnétique et un plasma. Dans ce transfert, les physiciens ont depuis longtemps identifié des modes qu'on pourrait dire résonants, qui sont les modes O (comme Ordinaires) et X (comme eXtraordinaires). La différence entre ces deux modes était mal comprise par les modèles physiques, sauf dans certains cas particuliers. Nous généralisons ces résultats dans un cas quelconque, en se ramenant à une équation différentielle ordinaire d'ordre 2 ayant un point régulier-singulier pour le mode X, et nous pouvons calculer le transfert d'énergie entre le champ magnétique et le plasma.



5 Invitation to preserver problems

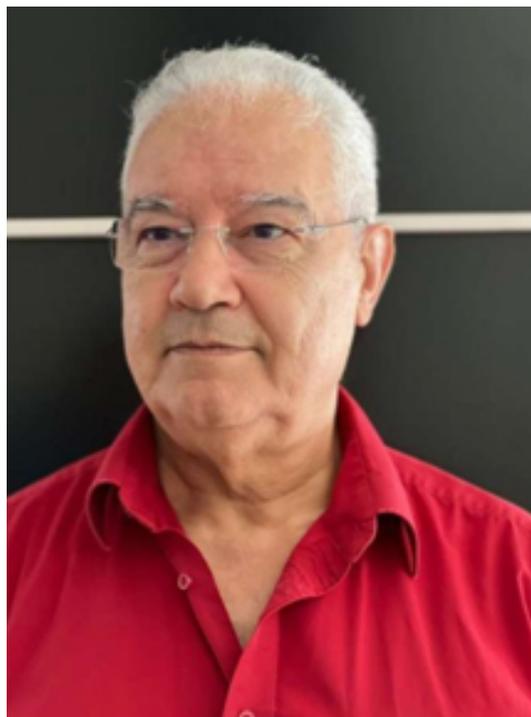
Mostafa Mbekhta¹

¹Université de Lille, France

Abstract

Preserver problems is an active research area in Matrix, Operator Theory and Banach Algebras. These problems involve certain linear (or not) transformations on spaces of matrices, operators or Banach algebras...

In this talk, I will give some concrete examples of preserver problems and some open questions and problems.



6 Dynamique des populations invasives et mouvement brownien branchant

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Abstract

Une espèce invasive est une espèce qui s'installe et se développe en dehors de son milieu d'origine. En l'absence de prédateurs, ou de concurrence, cette population déplacée va se reproduire librement, et envahir son environnement, en détruisant les écosystèmes originels. On peut citer de nombreux exemples, le frelon asiatique ou la tortue de Floride en France, la perche du Nil au Maroc, le crapeau-buffle et l'âne en Australie, etc. On propose de modéliser l'invasion de ces espèces à l'aide d'un processus aléatoire simple : le mouvement brownien branchant. Il s'agit d'un processus de particules se déplaçant au hasard de façon indépendante, et se divisant pour créer de nouvelles particules. Les propriétés en temps long de cet objet ont été très étudiées. On s'intéressera ici au comportement des particules extrêmes, afin de comprendre la dynamique d'invasion des populations.



7 Modéliser ou trouver le bon langage

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Abstract

Décrire objectivement un phénomène, pour prévoir son évolution, être déjà d'accord sur ce qui advient, n'est pas chose aisée. Plutôt que de nous pencher sur des phénomènes naturels complexes, nous nous intéresserons à des objets mathématiques simples mais évocateurs vous deviendrez capable de dessiner de beaux entrelacs, à la mode arabe ou celte, pour enluminer des manuscrits ou faire de jolies frises. Et la mathématique viendra à votre secours pour diminuer la complexité apparente du dessin et vous faire voir la simple structure d'un graphe qui se cache derrière.



8 Le Fabuleux destin de la racine carrée de 2

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Abstract

Au carrefour de la géométrie, de l'algèbre et de l'arithmétique, la racine carrée de 2 est l'une de ces constantes fondamentales que, à l'instar d'un nombre π ou du nombre d'or, l'on retrouve partout en mathématiques. C'est aussi un nombre qui, durant ses quatre mille ans d'histoire, a été utilisé aussi bien pour des questions esthétiques que pour vérifier la fiabilité des ordinateurs. Enfin, c'est un nombre qui, malgré la simplicité et l'ancienneté de sa définition, garde encore une part de mystère.



9 Perspectives de localité et approximations locales en analyse

Laurent Vivier¹

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Abstract

La conférence s'intéresse à la spécificité de la visualisation en analyse. On présentera un cadre didactique qui permettra de prendre en compte les aspects locaux, fondamentaux en analyse. Des exemples, proposés à des étudiants au début d'université, seront présentés et étudiés avec ce cadre. On montrera notamment différents phénomènes didactiques originaux.



Part II

Participants

1 Développement limité d'une fonction réelle à l'aide des matrices de Toeplitz

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Abstract

The limited development of a real function is the main tool for local approximation of a function by a polynomial, and also a very powerful tool in several fields of application. Before reaching the form described in the current definitions, the concept of limited development had to go through several non-linear steps and involve the use of various geometric techniques, analytical and algebraic developed by mathematicians of different civilizations from the early seventeenth century until the late nineteenth century. In this work, we discover a new matrix method for the limited development of a real function, which essentially uses Toeplitz matrices.

Keywords: Fonction réelle, Développement limité, Matrice de Toeplitz.

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2 THE DUALITY PROBLEM FOR THE CLASS OF ORDER WEAKLY DEMICOMPACT OPERATORS AND SOME PROPERTIES ON ORDER WEAKLY* DEMICOMPACT OPERATORS

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Abstract

In this paper, we investigate the duality problem for the class of order weakly demicompact operators. Precisely, we give a sufficient and necessary condition under which the order weak demicompactness of an operator implies the order weak demicompactness of its adjoint and conversely. Also, we introduce a new class of order weakly* demicompact operators. In addition, we establish some properties of this class of operators. As consequences, we obtain some characterizations of Banach lattices with order continuous norms or whose topological duals have order continuous norms.

Keywords: Duality problem, Banach lattice, Order continuous norm, Demicompact operator.

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3 Modeling of Tumor Growth and Drug Resistance in Gastrointestinal Stromal Tumor Metastases to the Liver

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Abstract

In this study, we examine mathematically and through numerical analysis a partial differential equation model for tumor growth that takes into account drug resistance. Specifically, we focus on modeling the growth and resistance to therapies of gastrointestinal stromal tumor metastases in the liver, and specifically the resistance to two tyrosine kinase inhibitor therapies (Imatinib and Sunitinib). Using medical images, we develop a spatial model of non-linear partial differential equations. This model accurately depicts the spatial progression of one particular patient's tumor. The first part of the paper proves the well-posedness of the model given certain conditions on the initial tumor. The second part presents numerical results from simulations and compares them to the clinical data of one specific patient for which we have complete treatment information, thus validating the model.

Keywords: Cancer Modeling, Tumor Growth, Drug Resistance, Numerical Simulations , PDEs.

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4 Exponential stabilization for a class of time delay bilinear systems of neutral type

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Abstract

This work addresses the exponential stabilisation question for bilinear time delay systems with neutral type, evolving on a real Hilbert space. Then, we consider the decomposition of the state space via the spectral properties of the systems to discuss the stabilization problem. The stabilisation of a such system reduces stabilising only its projection on a suitable finite dimensional subspace. Finally, an example is considered to illustrate the effectiveness of the obtained theoretical results.

Keywords: Non-homogenous systems, bilinear systems, Time delay, Neutral systems, Exponential stabilization.

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5 N -tuple sum analogues for Ramanujan-type congruences

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Abstract

In this talk we establish supercongruence relations for truncated N -tuple sums of basic hypergeometric series. As an application, we give double, triple, and quadruple sum analogues of some Ramanujan-type supercongruences.

6 A quantitative and qualitative study for some partial differential inclusions in α -norm

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Abstract

Let $(X, \|\cdot\|)$ be a Banach space. In this work, we investigate the existence of asymptotically almost periodic solutions of the following partial integro-differential inclusion:

$$\{ x'(t) \in Ax(t) + F(t, x(t)) \quad \text{for } t \geq 0, x(0) = x_0 \in X_\alpha,$$

The state variable $x(\cdot)$ takes values in the Banach space $(X_\alpha, \|\cdot\|_\alpha)$, which denotes the domain for the fractional power operator A^α equipped with the graph norm that will be described later. The operator $A : D(A) \subset X \mapsto X$ is the infinitesimal generator of an analytic semigroup $(\mathcal{T}(t))_{t \geq 0}$ defined on X . The multivalued function F is defined on $\mathbb{R}^+ \times X_\alpha$.

Our main interest is to investigate the existence of mild solutions and asymptotically almost periodic mild solutions for the system (1) where the forcing multivalued function F is defined on $\mathbb{R}^+ \times X_\alpha$. To achieve this, we employ a fixed point theorem that is based on a scale of Banach spaces. Finally, we provide an example to illustrate the abstract results.

Keywords: Differential inclusion, Mild solution, Asymptotically almost periodic solution, Analytic semigroup.

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7 Weighted Fractional Ostrowski, Trapezoid, and Grüss type inequalities on time scales

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Abstract

In this paper we first derive a weighted fractional Montgomery identity on time scales, and then establish Weighted Fractional Ostrowski, Trapezoid, and Grüss type inequalities on time scales, respectively. These results not only provide a generalization of the known results, but also give some other interesting inequalities on time scales as special cases.

Keywords: Montgomery identity, Ostrowski inequality, Trapezoid inequality, Grüss inequality, time scales, conformable fractional calculus.

8 Optimal Control of a General impulsive VS-EIAR Epidemics Models with Application to Covid-19

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Abstract

In this paper, we are interested in a VS-EIAR epidemiological model considering vaccinated individuals $\{V_i : i = 1, \dots, n\}$. The dynamic of the VS-EIAR model involves $n + 6$ ordinary differential equations that describe the evolution of vaccinated, susceptible, infected, exposed, asymptomatic and deaths people groups using vaccination doses and the treatment to the infected individuals. For this purpose, we use optimal control theory to control the dynamic of our considered epidemic model in a terminal optimal time τ^* by minimizing a well defined cost functional \mathcal{J} . Pontryagin's maximum principle (PMP) will be used to establish the existence and the uniqueness of such an optimal control time $(v^*(t), u^*(t), \tau^*)$ which at the cost functional \mathcal{J} reaches its minimum. Also, an impulsive VS-EIAR epidemic model will be taken into account with a special attention since the immigration or the travel of some people which make our study more applicable. A numerical simulation is given to show how can the theoretical study be applied.

Keywords: Optimal Control, Covid-19, VS-EIAR epidemic Model, Impulsive Epidemic Model, Mathematical Modeling, Ordinary differential equations.

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9 On the study of some nonlinear and noncoercive elliptic problem with Neumann boundary condition

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Abstract

In this work, we prove the existence of solutions in the sense of distributions for the anisotropic nonlinear Neumann problem:

$$\begin{cases} -\sum_{i=1}^N \frac{\partial}{\partial x_i} (b(|u|) |\frac{\partial u}{\partial x_i}|^{p_i-2} \frac{\partial u}{\partial x_i}) + |u|^{s_0-2} u + d(|u|) |\nabla u|^{p_i} = f & \text{in } \Omega, \\ \frac{\partial u}{\partial n_i} = 0 & \text{in } \partial\Omega, \end{cases}$$

where Ω is a bounded open subset in \mathbb{R}^N , ($N \geq 2$) with Lipschitz boundary $\partial\Omega$, $s_0 = \max(p_i)$, $\forall i = \{1, \dots, N\}$, $b(|\cdot|)$ is a positive decreasing function, $d(|\cdot|)$ is a continuous decreasing function such that $\frac{d(|\cdot|)}{b(|\cdot|)} \in L^1(\Omega) \cap L^\infty(\Omega)$ and $f \in L^m(\Omega)$. We prove the existence of solutions in sense of distributions for the data in $L^\infty(\Omega)$ and in $L^m(\Omega)$. Also, we conclude some regularity results.

Keywords: Anisotropic Sobolev spaces, Neumann Problem, Nonlinear elliptic problem, Solution in sense of distributions.

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10 Solvability of strongly nonlinear elliptic problems in Musielak-Orlicz spaces

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Abstract

In this paper we investigate the existence result of entropy solution for some nonlinear elliptic problems of Leray-Lions type associated to the equation $-\operatorname{div} a(x, u, \nabla u) = f(x) - \operatorname{div} F(u)$ in Ω , with large monotonicity condition in the setting of Musielak-Orlicz-Sobolev spaces and where the right hand side f belongs to $L^1(\Omega)$ and $F = (F_1, \dots, F_N)$ satisfies $F \in (C^0(\mathbb{R}))^N$.

Keywords: Elliptic problem; Entropy solutions; Musielak-Orlicz-Sobolev spaces, Compact imbedding, Δ_2 -condition.

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11 On the positive solutions of a nonlinear elliptic equation with a gradient term

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Abstract

In this paper, we are concerned with the study of the elliptic equation

$$\Delta_p u + \alpha u + \beta x \cdot \nabla u + |u|^{q-1} u = 0, \quad x \in \mathbb{R}^N,$$

where $p > 2$, $q > 1$, $N \geq 1$, $\alpha > 0$ and $\beta > 0$.

We show the existence of radial solutions and we present the asymptotic behavior of positive solutions near infinity. More precisely, we prove under some assumptions that there exists a positive solution u which has the following behavior near infinity,

$$u(r) \underset{+\infty}{\sim} \left(N - p - \frac{\alpha}{\beta} (p-1) \right)^{\frac{1}{q+1-p}} \left(\frac{\alpha}{\beta} \right)^{\frac{p-1}{q+1-p}} r^{-\alpha/\beta}.$$

Keywords: Nonlinear elliptic equation, Radial self-similar solution, Global existence, Energy Function, Asymptotic behavior, Equilibrium point, Nonlinear dynamical systems.

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12 Existence and uniqueness of a mild solution for a class of the fractional evolution equation With nonlocal condition involving φ -Riemann Liouville fractional derivative

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Abstract

In this paper, by using the fractional power of operators and theory fixed point theorems, we discuss Existence and uniqueness of mild solution to initial value problems for fractional semilinear evolution equations with compact semigroup in Banach spaces with nonlocal conditions. In particular, we derive the form of fundamental solution in terms of semigroup induced by resolvent and φ -Riemann-Liouville fractional derivatives. These results generalize previous works where the classical Riemann-Liouville fractional derivative is considered. In the end, we give an example to illustrate the applications of the abstract results.

Keywords: Mild solution, Fractional evolution equation, φ -Riemann-Liouville fractional derivative, Measure of noncompactness, Darbo-Sadovskii's fixed point theorem.

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13 CONFORMABLE FRACTIONAL DIFFERENTIABLE FUNCTION AND APPLICATIONS

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Abstract

In this paper, we generalize the notion of the fixed point and we prove a fixed point theorem for a contraction in \mathbf{R} using the definition of fractional derivative given by [1].

Keywords: α -fixed point, Fractional derivative, Fractional integral.

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14 On Pseudo-Hermitian Quadratic Nilpotent Lie Algebras

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Abstract

We study nilpotent Lie algebras endowed with a complex structure and a quadratic structure which is pseudo-Hermitian for the given complex structure. We propose several methods to construct such Lie algebras and use a method of double extension by a plane to get an inductive description of all of them. Such a method let us give a complete classification of nilpotent quadratic Lie algebras where the metric is Lorentz-Hermitian and to obtain a complete classification of pseudo-Hermitian quadratic Lie algebras up to dimension 8 and their inequivalent pseudo-Hermitian metrics.

Keywords: Pseudo-Hermitian Lie algebra, quadratic Lie algebra, quadratic double extension.

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15 Existence and behavior of singular solutions of a nonlinear boundary value problem with the p-Laplacian operator

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Abstract

In this paper we study singular radial solutions of the following nonlinear boundary value problem

$$(P) \begin{cases} (|u'|^{p-2}u')' + \frac{N-1}{r}|u'|^{p-2}u' + r^{l_1}|u|^{q_1-1}u + r^{l_2}|u|^{q_2-1}u = 0, & r > 0 \\ u(0) = a, \quad \lim_{r \rightarrow 0} r^{\frac{N-1}{p-1}}u'(r) = 0, \end{cases}$$

where $N > p > 2$, $q_2 > q_1 > p - 1$, $-p < l_2 < l_1 < -1$ and $a > 0$.

We establish the global existence of positive solutions and we give a classification of radial solutions of the above equation. Moreover, under some assumptions, we show that the positive solution has the following asymptotic behavior near the origin and infinity.

Keywords: Boundary value problem; singular solutions; global existence; positive solutions; asymptotic behavior; energy methods.

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16 Infinitely many solutions for a class of fractional equations via variant fountain theorems

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Abstract

This article is concerned with a class of fractional type equation involving an anisotropic operator and potential of the form

$$(-\Delta_x)^s u - \Delta_y u + \Phi(x, y)u = g(x, y, u), \quad (x, y) \in \mathbb{R}^n \times \mathbb{R}^m.$$

By means of the variational method and the variant fountain theorems, we investigate the existence of infinitely many high or small energy solutions without the usual assumption of coerciveness on the potential Φ in the different cases when the nonlinear term is either asymptotically linear or superquadratic growth.

Keywords: Potential BO-ZK equation, Infinitely many solutions, Variational method.

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17 Existence result of entropy solution for degenerate elliptic problem without monotonicity condition in Generalized Sobolev spaces

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Abstract

In the present paper we prove some existence results of entropy solution for nonlinear degenerate elliptic problems of the form $Au + h(x, u) = f$, in Musielak-Orlicz-Sobolev spaces, where $A(u) = \text{div}(a(x, u, \nabla u))$ is a Leray-Lions operator defined from the Musielak-Orlicz-Sobolev spaces $W_0^1 L_\varphi(\Omega)$ into its dual. The right hand side $f \in L^1(\Omega)$, and no monotonicity strict condition is assumed on the function $a(x, s, \xi)$, The tool we use to overcome this difficulty is to investigate some techniques introduced by Minty's lemma.

Keywords: Elliptic problem; Entropy solutions; Weighted Orlicz-Sobolev spaces, Musielak spaces, Monotonicity condition.

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18 An approach method with application of Covid data using variational spline functions

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Abstract

The Covid-19 pandemic has affected almost all the countries of the world in the space of four months. In mid-April 2020, the epidemic had already absorbed, even saturated, the hospital capacities of many countries, causing more than 150000 deaths worldwide and seriously affecting the health of at least half a million people. The circulation time of the virus therefore proved to be much shorter than the various reaction times of human institutions, whether hospital, scientific or political. The duration of the latency period, the incubation time, the duration of contagiousness, the duration of hospitalization, the delays between infection and death are all important parameters that must be able to be estimated and integrated into the modeling in order to understand the dynamics of a Covid-19. In addition, one of the difficulties encountered in the study of this pandemic comes from the number of infected individuals because there are people who contract the virus without declaring symptoms, while becoming contagious.

In [3] we present a new method which allows to adjust the statistical difficulty when the statistical series are spliced. Hence, we have studied the scope of different splicing methods in the literature. We have presented an approximation method for statistical splicing of economic data by using smoothing quadratic splines. Finally, we have shown the effectiveness of our method by presenting a complete data of Gross Domestic Product for Venezuela by productive economic activity from 1950 to 2005, expressed at prices of the base year of 1997, also by showing the results of some data of Morocco for different economics activities such as the GDP (Gross Domestic Product), the agriculture, the trade and the electricity generation from petroleum sources of Morocco between 1971 and 2015. Note that this work has been presented in the first version of this conference "ICOSMEM'20".

Therefore, the main goal of this investigation is to approximate the Covid data using the variational method approach, specially by the variational splines, for more details, reference ([1]) can be consulted, and for research works in the same line, reference ([2]) can also be consulted. While as a real application, as we have done in [3], we present a suitable study to Covid data of Morocco and an analysis of the inconveniences on the Moroccan economy.

The information was obtained from official sources, specifically from data bases published by Moroccan Ministry of Health to ensure the highest quality for this study.

Keywords: Covid data, Variational spline, Approximation methods, Application to Economic.

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19 A bilevel problem based on fractional-space derivative for image denoising

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Abstract

In this work we propose a bilevel approach to deal with image denoising problem. An optimal choice of the fractional order derivative is carried out with the lower problem, while the upper one deals with denoising process. An algorithm is also proposed based on the Primal-Dual approach. Numerical results are based on finite differences, which show remarkable results compared to some well-known competitive models.

Keywords: fractional derivative, bilevel problem, image denoising, Primal-Dual.

20 Fractional Output Stabilization of Distributed Bilinear Systems with a Time Delay

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Abstract

This research involves studying fractional spatial output stabilization for bilinear systems with a time delay. Hence, we give an expression of control that depends on delay to show the strong, weak, and exponential stabilization of the fractional output under some sufficient conditions. Finally, the computational simulations are taken to attempt the validity of the reached theoretical results.

Keywords: Stabilization, System with a time delay, Fractional output.

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21 Temporal Evolution of Tumor Growth: A Mathematical Modeling Approach-Simulation and identification of parameters

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Abstract

In this study, we present a mathematical model of tumor growth in the case of liver metastases of gastrointestinal stromal tumors (GIST) using ordinary differential equations (ODEs). The model takes into account the impact of cytotoxic treatment and is a coupled ODE system. The first part of the study involves numerical simulation of the model through classical methods, while the second part focuses on optimizing the model parameters through the method of least squares based on clinical data to increase accuracy and align with scan results. Finally, the study explores the emergence of treatment resistance and the timing of when a tumor begins to resist treatment.

Keywords: Numerical method, Modeling, GIST, Tumor Growth, OED, Least square method, Emetgence

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22 Square root problem and Subnormal Aluthge transforms

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Abstract

For a non negative measure μ with p atoms, we study the relation between the Square Root Problem of μ and the problem of subnormality of \tilde{W}_μ the Aluthge transform of the associated unilateral weighted shift.

We use an approach based on uniquely represented elements in the support of $\mu * \mu$. We first show that if \tilde{W}_μ is subnormal, then $2p - 1 \leq \text{card}(\text{supp}(\mu * \mu)) \leq \lceil \frac{(p-1)^2+6}{2} \rceil$. We rewrite several results known for finitely atomic measure having at most five atoms and give a complete solution for measures six atoms.

Keywords: Finitely atomic measures, support of multiplicative convolution, subnormal Aluthge transform of weighted shifts, the Square Root Problem for measure.

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23 Commutative Sub-algebras in Four-Dimensional Absolute Valued Algebras

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Abstract

The aim of this talk is to construct, by algebraic methods, some new class of four-dimensional absolute valued algebras having two different commutative sub-algebras of dimension two [2], [7] and [5]. These new algebras contain a nonzero omnipresent idempotent. Furthermore, we classify all four-dimensional absolute valued algebras containing a nonzero idempotent commuting with all idempotents [3], [1] and [4], in this classification, we conclude such an algebra contains at least two different commutative sub-algebras of dimension two. Note that there exists a four dimensional absolute valued algebra containing no sub-algebra of dimension two [6], which means that, the problem of classifying all four-dimensional absolute valued algebras seems still to be open.

Keywords: Absolute valued algebra, division algebra, commutative algebra, omnipresent idempotent.

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24 On four-dimensional absolute valued algebras with left omnipresent unit

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Abstract

A classification of all four-dimensional absolute valued algebras with left omnipresent unit is given [1] and [2]. We construct, by algebraic methods all four-dimensional absolute valued algebras with left omnipresent unit. These new algebras contain at least one sub-algebra of dimension two [4], note that there exists a four-dimensional absolute valued algebra with left unit containing no sub-algebra of dimension two [3].

Keywords: Absolute valued algebra, Division algebra, left unit, omnipresent idempotent.

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25 A Common fixed point results for multi-valued mappings in Hausdorff modular fuzzy b -metric spaces

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Abstract

The modular fuzzy b -metric space is defined in this study, and we are interested in proving a general common fixed point theorem for a pair univalued mappings in modular fuzzy b -metric spaces. The findings in this work generalize the findings in [1] and produce additional specific findings that are supported by examples. An application to prove the existence of an integral equation's solution is shown to demonstrate the importance of our result.

Keywords: Fuzzy metric space, modular b -metric space, modular fuzzy b -metric space, t-norm, fixed point.

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26 A novel quantum information based computational approximate solution of the Schrödinger partial differential equation

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Abstract

In this work, we solve the Schrödinger partial differential equation

$$i\hbar \frac{\partial \phi}{\partial t}(t, X) + \frac{\hbar^2}{2m} \Delta \phi(t, X) - V_0(X)\phi(t, X) - V_1 *_t \phi(t, X) = f(t, X)$$

for all $(t, X) \in [0, +\infty[\times \Omega$, with the initial conditions $\phi(0, X) = \phi_0(X)$ and the following boundary conditions

$$\phi(t, \xi) + \frac{\hbar^2}{2m} \frac{\partial \phi}{\partial \vec{n}_\xi}(t, \xi) = g(t, \xi), \quad (t, \xi) \in [0, +\infty[\times \partial\Omega$$

We use the frequency-domain method to solve the considered problem, as a partial differential equation with non-homogeneous boundary conditions. The method employs the Fourier transform Discretization (FTD) and consists of two stages. In the first stage the equations are transformed into an equivalent problem for the frequency variables. The numerical solutions of this problem are approximated using a Galerkin projection based on the higher-order Spline finite element method. In the second stage a several quadrature procedure are used for the calculation of the solution of the inverse Fourier transform, and then we gives a comparison report between the various numerical computations of this integral. The frequency domain method avoids the discretization of the time variable in the considered problem. Finally, several test examples are presented to verify high accuracy, effectiveness, good resolution properties for smooth and discontinuous solutions and plots of field of displacements of the waves.

Keywords: Quantum information, Schrödinger PDE, frequency-domain approach, Spline finite element analysis.

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27 Investigation of Ant Colony with Levy Flight Techniques for Stochastic Combinatorial Optimization Problems

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Abstract

The demand for efficient solutions to optimization problems with uncertain and stochastic data is increasing. Stochastic Combinatorial Optimization Problems (SCOPs) involve partially unknown information about problem data with a known probability distribution [1]. Although classical methods based on mathematical and dynamic programming can provide optimal solutions, they may not be feasible for large problem instances due to their computational complexity. To address this issue, the Levy flight method is often used to balance the search space and speed for global optimization. In this study, we propose an enhanced ant colony algorithm that utilizes the Levy flight mechanism to solve SCOPs, specifically the probabilistic traveling salesman problem (PTSP) where the presence of customers is modeled stochastically [2]. This approach enables some ants to take long jumps using the Levy distribution to avoid local optima situations [3]. Simulation results demonstrate that the proposed algorithm with Levy flight outperforms the conventional ant colony algorithm in solving the probabilistic traveling salesman problem. Overall, this study provides a promising approach to enhancing the performance of ant colony algorithm in solving stochastic combinatorial optimization problems.

Keywords: Combinatorial optimization, Stochastic, Levy Flight, Ant Colony, Probabilistic traveling salesman problem.

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28 On rings with Property (*a.c.*) and their extensions

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Abstract

Let R be an associative unital ring and let σ be an endomorphism of R . R is said to have the Property (*a.c.*) if for each finitely generated ideal I of R , there exists an element $c \in R$ such that $r_R(I) = r_R(RcR)$ [4]. The aim of this communication is to present some rings having the Property (*a.c.*), as well as investigating this property over σ -skew quasi-Armendariz rings.

Keywords: Annihilator condition, p.q.-Baer rings, σ -skew quasi-Armendariz rings.

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29 Entropy solutions for some nonlinear and noncoercive parabolic problems in the anisotropic Sobolev spaces

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Abstract

In this paper, we investigate the existence of an entropy solution for the following nonlinear and non-coercive parabolic problem

$$\frac{\partial u}{\partial t} + Au + \nu|u|^{s-1}u = \sum_{i=1}^N F_i(x, t, u) \quad \text{in } Q_T,$$

where A is a non-coercive operator of Leray-Lions type acted from the anisotropic parabolic space $L^{\vec{p}}(0, T; W_0^{1, \vec{p}}(\Omega))$ into its dual, with $F_i(x, t, \cdot)$ is a nonlinear term having a singularity term on x .

Keywords: Quasilinear parabolic equations, Hardy potential, non-coercive problems, entropy solutions.

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30 Renormalized periodic solutions for a nonlinear parabolic equation involving $p(x)$ -growth conditions with irregular data

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Abstract

This work aims to examine the existence of periodic solutions to a nonlinear parabolic equation driven by $p(x)$ -Laplacian operator with irregular data. By assuming that the data belongs in the Lebesgue space L^1 , we establish the existence of a renormalized periodic solution to the studied model. The functional framework involves generalized Lebesgue and Sobolev spaces with variable exponents.

Keywords: L^1 data, nonlinear parabolic equation, $p(x)$ -Laplacian operator, variable exponents, Renormalized periodic solution.

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31 Notes on complex symmetric Toeplitz operators on Hardy space and truncated Toeplitz operators

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Abstract

In this paper, we characterize the complex symmetric Toeplitz operator on the Hardy space via a kind of canonical conjugation on H^2 introduced by M.S. Ferreira in [1]. In model space equipped with a conjugation, we prove some results for truncated Toeplitz operators in relationship with complex symmetry according to [3].

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32 An efficient algorithm based on particle swarm optimization for solving the Cauchy problem for Helmholtz equation

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Abstract

In this work, we consider a Cauchy problem associated with the Helmholtz equation. This problem consists in recovering the missing conditions on the inaccessible part of the boundary from over-specified measurements available on the accessible one. To solve this ill-posed problem, we use an optimization approach based on particle swarm optimization (PSO) in combination with the finite element method (FEM). Numerical simulations for an irregular domain show the feasibility and effectiveness of this algorithm and demonstrate that our approach can recover a close estimate of the unknown Cauchy data.

Keywords: Inverse Problem, Particle Swarm Optimization, Data completion problem, Helmholtz equation.

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33 Simple and fast convergent procedure to estimate recursive path analysis model

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Abstract

The purpose of statistics is to describe the phenomena studied and to give forecasts, and make decisions. To describe the phenomena studied using methods and techniques such as linear regression, principal component analysis (PCA), canonical analysis,....

In this paper we will focus on the method called Path analysis (PA), PA discovered by Wright (1921, 1923, 1934) is a statistical technique used to examine cause-and-effect relations between a set of observed variables. On the one hand, PA can be seen as a straightforward extension of multiple regression. On the other hand, it is the starting point of structural equation modelling (SEM). PA is performed upon five steps: specification, identification, estimation, testing, and modification (Schumacker and Lomax 2004). The estimation step appears as the core of the modelling process. It consists on finding values for the unknown parameters by minimizing a given criterion. BFGS procedure is classically used for the estimation of the parameters of a recursive Path Analysis model. In practice, BFGS does not present any problem of convergence. However, to date, no proof of its convergence is available. The present paper introduces an alternative procedure and establishes its convergence properties.

Keywords: Path analysis, Correlation matrix, Finite iterative method, Unweighted least squares, Generalized least squares.

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34 Exponential and weak stabilization of distributed perturbed semilinear systems

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Abstract

This paper deals with the problem of feedback stabilisation for a class of distributed semilinear systems evolving on a real Hilbert state space, under external perturbation given by nonlinear operator. Firstly, we discuss the existence and uniqueness of the global mild solution of the considered systems. Moreover, we give sufficient conditions to guarantee exponential, strong and weak stabilizations of such systems. Then, we characterize controls, that stabilizes the state, and minimizes a given performance cost. Finally, the obtained results are illustrated by examples and numerical simulations.

Keywords: Semilinear systems, Optimal feedback control, Exponential stabilization.

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35 SEMI-COMPACTNESS OF NULL ALMOST L-WEAKLY COMPACT OPERATORS ON BANACH LATTICES

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Abstract

The subject of this work is mainly concerned with the functional analysis, more precisely, in the field of the theory of positive operators on Banach lattices. And the aim is to study the relation between the class of semi-compact and the class of Null almost L-weakly compact operators and we give some interesting consequences. We investigate conditions on a pair of Banach lattices E and F that tells us when positive semi-compact operator is Null almost L-weakly compact and conversely.

Keywords: Semi-compact, Null almost L-weakly compact, order continuous, Banach lattice.

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36 Unilateral problem associated to some noncoercive elliptic equation in anisotropic Sobolev spaces

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Abstract

This paper is devoted to study existence of unilateral problem associated to the degenerated elliptic equation, whose prototype is given by

$$\begin{cases} Au + g(x, u) = f(x) - \operatorname{div} F(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

in the anisotropic Sobolev space, where Ω is a bounded open subset set of \mathbb{R}^N ($N \geq 2$), where $f \in L^1(\Omega)$ and $F(x, u)$ satisfying only some growth condition. We show the existence of entropy solutions and we will conclude some regularity results.

Keywords: Anisotropic Sobolev spaces, non-coercive quasilinear elliptic equation, unilateral problems, entropy solutions.

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37 On multivalued probabilistic ψ -contractions involving orbits in b-Menger spaces

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Abstract

In this paper, we give a new approach of some well-known fixed point theorems for multivalued probabilistic ψ -contraction in b-Menger space by using the boundedness of the orbits. As an application of these results, we also obtain the corresponding fixed point theorem in metric spaces. Our results improve and generalize the results given by Hadžić [6] and Fang [4].

Keywords: Fixed points, b-Menger spaces, multivalued ψ -contraction.

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38 A critical Steklov-type problem driven by a $p(x)$ -Laplace operator

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Abstract

In this work, we consider some class of Steklov-type problems involving the $p(x)$ -Laplace operator with variable exponents. Using variational methods combined with the Mountain pass lemma, we prove some existence and multiplicity results for the given problem in an appropriate functions spaces. The main results of this paper improve and generalize the previous ones introduced in the literature.

Keywords: PDE, $p(x)$ -Laplace operator, Steklov problem, variational methods.

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39 Unilateral problems having two lower order terms and measure data in Musielak-Orlicz spaces

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Abstract

This paper is concerned with the existence results for nonlinear unilateral elliptic problems having two lower order terms and measure data in Musielak-Orlicz spaces, where the Musielak function verify the log-Hölder continuity condition.

Keywords: Musielak-Orlicz-Sobolev spaces, elliptic problem, renormalized solutions, truncations.

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40 Strongly nonlinear elliptic problem with L^1 -data in the setting of Musielak-spaces

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Abstract

The aim of this paper is to investigate an existence of renormalized solutions for some boundary value elliptic problem of the form $-\operatorname{div}(a(x, u, \nabla u) + \Phi(x, u)) + g(x, u, \nabla u) = f$ in Ω , in the framework of Musielak-Orlicz spaces, where the term Φ satisfies the natural growth condition, the function g has a natural growth with respect to its third argument and without sign condition, no Δ_2 -condition is assumed on the Musielak function, and $f \in L^1(\Omega)$.

Keywords: Musielak-Orlicz-Sobolev spaces, elliptic problem, renormalized solutions, truncations.

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41 Weak periodic solution for strongly nonlinear parabolic problems

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Abstract

This work deals with an existence result of a weak periodic solutions for non-linear parabolic equations of the type

$$\begin{cases} \frac{\partial u}{\partial t} + \Delta u + H(x, t, u, \nabla u) = f & \text{in } Q = \Omega \times]0, T[, \\ u(x, 0) = u(x, T) & \text{in } \Omega, \\ -\frac{\partial u}{\partial \nu} = \beta(x, t)u + h(x, t, u) & \text{on } \Sigma = \partial\Omega \times]0, T[, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ ($N \geq 2$) bounded open domain with smooth boundary denoted by $\partial\Omega$. We assume that

- f is a periodic function such that $f \in L^2(Q)$.
- β is a periodic positive continuous and bounded function.
- $h : \Sigma \times \mathbb{R} \mapsto \mathbb{R}$ is a Carathéodory function periodic in time, $s \mapsto h(x, t, s)$ is nondecreasing for a.e $(x, t) \in \Sigma$, $h(x, t, s)s \geq 0$ and $|h(x, t, s)| \leq \xi(x, t) + |s|$ where $\xi \in L^2(\Sigma)$.
- $H : Q \times \mathbb{R} \times \mathbb{R}^N \rightarrow \mathbb{R}$ is a Carathéodory function such that $H(x, t, s, \xi) \in L^1(Q)$ $\forall s \in \mathbb{R}, \forall \xi \in \mathbb{R}^N$ and a.e $(x, t) \in Q$.
- $|H(x, t, s, \xi)| \leq g(s)|\xi|^2$ a.e $(x, t) \in Q, \forall s \in \mathbb{R}, \forall \xi \in \mathbb{R}^N$, where $g : \mathbb{R} \rightarrow \mathbb{R}^+$ is a continuous function and $g \in L^1(\mathbb{R})$.

Keywords: Weak Periodic solution , parabolic equation , nonlinear boundary conditions.

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42 Multiplicity of solutions for a bi-nonlocal elliptic problem involving the $p(x)$ -Laplacian

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Abstract

In this article, we deal with a class of $p(x)$ -Kirchhoff equation where the non-linearity has non-standard growth and contains a bi-non-local term. Using variational methods, we prove the existence and multiplicity of solutions for the given problem under certain assumptions.

Keywords: $p(x)$ -Laplacian operator, Variational methods, Kirchhoff problem, Bi-nonlocal term.

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43 Using the Sherman-Morrison-Woodbury Inversion Formula to solve nearly penta-diagonal Toeplitz linear systems

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Abstract

In this work, we are interested in solving nearly penta-diagonal Toeplitz linear systems $Tx = b$ where T is a matrix of size $n \times n$ given by :

$$T = \begin{bmatrix} t_0 & t_{-1} & t_{-2} & 0 & \dots & 0 & t_{2-n} & t_{1-n} \\ t_1 & t_0 & t_{-1} & t_{-2} & \ddots & \ddots & 0 & t_{2-n} \\ t_2 & t_1 & t_0 & t_{-1} & t_{-2} & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & t_2 & t_1 & t_0 & t_{-1} & t_{-2} \\ t_{n-2} & 0 & \ddots & \ddots & t_2 & t_1 & t_0 & t_{-1} \\ t_{n-1} & t_{n-2} & 0 & \dots & 0 & t_2 & t_1 & t_0 \end{bmatrix}, \quad x \text{ and } b \in \mathbb{R}^{n,1}$$

We propose a fast numerical algorithm based on the use of the Sherman–Morrison–Woodbury inversion formula. By exploiting the special matrix structure of T , we propose a new decomposition form of the coefficient matrix. Then, the application of the well-known Sherman–Morrison–Woodbury inversion formula, we compute the inverse of the considered matrix which allow to solve the system. Numerical results for different standard instances are provided to illustrate the accuracy and the efficiency of the proposed algorithm.

Keywords: Nearly penta-diagonal Toeplitz matrix, linear system, Sherman–Morrison–Woodbury formula.

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44 On the study of some noncoercive elliptic equations in anisotropic weighted Sobolev spaces

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Abstract

In this work, we consider some nonlinear and non-coercive elliptic Dirichlet problem

$$\begin{cases} Au + d(|u|)|\nabla u|^p \omega_0 = f(x, u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

in the anisotropic weighted Sobolev spaces. We study the existence and regularity of renormalized solutions for this elliptic equation. Also we will conclude some regularity results.

Keywords: Anisotropic Weighted Sobolev spaces, elliptic equation, renormalized solutions.

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45 Management of Moroccan Fisheries with Variable Market Price

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Abstract

We present a dynamical model of fishery describing the time evolution of the fish stock, the fishing effort with a variable price, and the market price is fixed by the gap between the supply and the demand. The three-dimensional model considers a nonlinear harvesting function. Assuming two-time scales, we use "aggregation of variables methods" in order to derive a reduced model governing fish density and fishing efforts at a slow time scale. This reduced model is analyzed and according to parameters values, three main cases can occur: the first one is locally asymptotically stable; the fish population becomes extinct, fishing efforts tend to a positive constant value and the price tends to infinity when approaching equilibrium, this case corresponds to over-exploitation leading to fish extinction, the second case corresponds to stable fishery equilibrium, and the third case corresponds to the coexistence of two strictly positive equilibria, the first one is a saddle and the second is stable.

Keywords: Demand function, Stability, Variable price.

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46 On a p -fractional Kirchhoff-type equation with critical exponent

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Abstract

This work deals with a p -fractional Kirchhoff type equation, for which the existence and nonexistence of solutions is proved by employing mountain pass theorem with Cerami condition and some technical lemmas, especially a variant of Lions lemma inspired by Ramos. More concretely, transforming the equation into an equivalent system, we show the existence of at least one solution or two solutions without the well-known Ambrosetti-Rabinowitz (AR) condition.

Keywords: Fractional p -Laplacian operator, Kirchhoff-type equation, Critical exponent.

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47 Speed Up The Convergence rate Via Tensor Extrapolation Methods

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Abstract

It's well-known that vector extrapolation methods such as Minimal Polynomial Extrapolation method (MPE) and Reduced Rank Extrapolation method (RRE) are widely used in scientific computation. Our purpose in this work is to present a generalization of these methods for tensor sequences. The proposed methods involve only the terms of sequences that result from iterative methods. Our approach is based on the notion of the n-mode minimal polynomial of a matrix with respect to a tensor. This polynomial will be used, through the iterative solution of some tensor linear systems, to introduce the tensor version of MPE and RRE. The implementation of these methods on some sequences of tensors confirms the effectiveness and applicability of our approach.

Keywords: Multilinear algebra, N-mode product, Tensor Extrapolation method, Convergence acceleration..

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48 Study of some non-coercive quasilinear parabolic problem in anisotropic Sobolev space

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Abstract

This paper is devoted to investigate the existence of renormalized solutions for the following quasilinear parabolic problem :

$$\begin{cases} u_t + Au = f(x, t, u, \nabla u) & \text{in } \Omega \times (0, T) = Q_T, \\ u = 0 & \text{on } S_T \\ u(0, x) = u_0(x) & \text{in } \Omega; \end{cases}$$

in the parabolic anisotropic Sobolev space. Where the term on the right-hand side $f(x, t, s, \xi)$ verify only some growth condition. Also we conclude some regularity results.

Keywords: Quasilinear parabolic equations, non-coercive problems, renormalized solutions.

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49 Adjusting support in CSRBF function interpolation for BEMD decomposition

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Abstract

Several time-frequency analysis methods have been developed, including the Fourier transform and the wavelet transform. In the case of non-stationary and non-linear signals, the previous techniques of time-frequency analysis remain unsuitable for this type of signal, despite their further developments. The bidimensional empirical mode decomposition (BEMD) [4], is a new time-frequency analysis method invented to overcome this problem. This decomposition enables the extraction of structures at different scales and frequencies, including amplitude and frequency modulations.

The major obstacle to this decomposition is the computational complexity, most of this calculation is done in the extremum interpolation phase. Several versions of the BEMD developed with different interpolation functions to minimize the cost and maintain the quality of this decomposition. One of the methods that show good results in terms of computational complexity and maintaining the quality of the decomposition is the BEMD with Compactly Supported Radial Basis Functions (BEMD- CSRBF) [2]. Despite the effectiveness of CSRBF functions, especially in terms of computational complexity, the choice of support size for these functions plays an important role in the quality of the BEMD, in particular that this decomposition is iterative, at each iteration we get a different number of extrema and a different distribution in space.

In this context, we will present in this article a study on the influence of the support size of the compactly supported radial Basis functions (Wendland functions [3]) on the BEMD, either in terms of computational complexity or decomposition quality. Finally, we propose a method to adjust the size of the support during the algorithm.

Keywords: Time-frequency analysis, Decomposition BEMD, CSRBF functions, Wendland functions

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50 On the asymptotic behavior of radial solutions to quasilinear elliptic equations with a Hardy potential

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Abstract

In this work, we are interested in studying the quasilinear elliptic equation with a Hardy potential.

$$\operatorname{div}(|x|^\alpha |\nabla u|^{p-2} \nabla u) + \frac{\mu}{|x|^{p-\alpha}} |u|^{p-2} u = 0, \quad x \in \mathbb{R}^N - \{0\},$$

where $N \geq 1; p > 2; q > 1; \alpha \in \mathbb{R}$ and $\mu \in \mathbb{R}^*$. In the first part, we show the existence of radial solutions in the case $\alpha = 0$. In the second part, we wrote the radial solutions in the form of the first order differential system

$$\begin{cases} x' = ax + b\Phi_q(y) \\ y' = c\Phi_p(x) + dy, \end{cases}$$

where $a, b, c, d \in \mathbb{R}$ and $\Phi_p(s) = |s|^{p-2}s$ for all $s \in \mathbb{R}$, which can be solved by the eigenvalues of the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, this means that the roots of the characteristic equation $(\lambda - a)(\lambda - d) - bc = 0$. In this section, the characteristic equation of the autonomous planar semilinear differential system is introduced, and the asymptotic behavior of its solutions is established by the roots of the characteristic equation.

Keywords: Quasilinear, elliptic equation, radial self-similar solution, Hardy potential, p-Laplacian, existence and uniqueness, energy function, asymptotic behavior, semi linear system, characteristic equation, eigenvalues, eigenvector, equilibrium point.

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51 Approximation in some spaces of analytic functions

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Abstract

We give an elementary proof of the approximation theorem established by A. Matheson for λ_α analytic Lipschitz spaces. Our approach allows us to extend this approximation theorem to superharmonically weighted Dirichlet spaces (including standard Dirichlet spaces) and to some analytic Besov spaces. As a consequence, we give a complete description of closed ideals for some analytic function algebras.

Keywords: Lipschitz spaces, Superharmonically weighted Dirichlet spaces, analytic Besov spaces, closed ideals.

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52 A fixed point approach to stability for a Cauchy-Jensen type functional equation in n-Banach space

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Abstract

The aim of this work is to provide and study the applications of the fixed point theorem (using the Brzdęk fixed point approach) to prove a new type of stability results of a Cauchy-Jensen type functional equation in n-Banach space.

Keywords: fixed point, 2-Banach space, n-Banach space, Cauchy-Jensen functional equation, stability.

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53 A Metaheuristic Approach to Improve Consistency of the Pairwise Matrix in AHP

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Abstract

In this paper, we are interested in modifying inconsistent pairwise comparison matrix which is a critical step in the AHP methodology, where decision maker's have to improve consistency by revising the process. To this end, we propose an improved genetic algorithm (GA) to allow decision maker's to find an appropriate matrix and adjust the consistency of their judgment without loss of original comparison matrix. Numerical results with different dimensions of matrices taken randomly show the effectiveness of these strategy to improve and identify the consistency of pairwise matrix which mean that GAs are a very good tool to generate consistent pairwise comparison matrices with different number of criteria.

Keywords: Genetic Algorithm, Pairwise Matrix, Analytic Hierarchy Process, Decision theory, Consistency.

54 On the Periodic Solutions for a class of Partial Differential Equation with Delay

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Abstract

Trough this work we investigate the periodicity of solutions for a class of partial differential equations with infinite delay of the form $w'(t) = Lw(t) + D(wt) + H(t)$. We suppose that the operator $(L, D(L))$ is generally nondensely defined operator and verifies the Hille-Yosida condition. Using the theory of perturbation of semi-Fredholm operators, we propose, when the phase space is a fading memory space, some sufficient conditions on the linear operators L and D to guarantee the periodicity of solutions of this class of partial differential equations from bounded ones on the positive real halfline. In addition, we consider the case where the operator L is a sum of two operators, the first one verifies the Hille-Yosida condition and the second one is a bounded linear operator. In this case, we give in the both situations of fading and uniformfading memory space, more sufficient conditions to derive periodic solutions from bounded ones. All this, without considering neither the compactness nor the exponential stability of the semigroup generated by the part of L on the closure of it's domain. At the end, an application with numerical

Keywords: Semigroup, Hille-Yosida condition, integral solutions, semi-Fredholm operators, Poincaré map, periodic solution, Simulation.

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55 Exponential stabilization for a semi-linear systems with discrete multi-delays

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Abstract

The multi-delays system has been studied by many researchers using several methods and approaches such as the Lyapunov method, for more details on this subject, we refer to [3,4] and references therein. This class of equations are initiated in control theory due to their simple forms and also to their applicability in many and different real-life problems such as biology, economics, engineering, chemistry, for more details about this subject the reader can see for example [5] and references therein. In this paper we investigate the stability for a semi-linear systems with discrete multi-delays on a Hilbert state space. Firstly, we discuss the well posedness of mild solutions of the considered systems. Secondly, some sufficient conditions are given to guarantee the feedback stabilization for the semi-linear systems. The stabilization results are given in term of observation estimates. At the end, examples with numerical simulations are given to show the applicability of our theoretical results.

Keywords: Feedback stabilization, semi-linear, multi-delays.

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56 Central finite volume methods for nonlocal traffic flow models with arrhenius-type look-ahead rules

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Abstract

In this paper, we describe a central finite volume method and apply it to a new class of traffic flow model with an Arrhenius-type look-ahead interaction. These models can be written as a scalar conservation law with a nonlocal flux. The proposed scheme is an extension of the Nessyahu-Tadmor non-oscillatory central scheme, which belongs to a class of projection evolution finite volume methods. We demonstrate the robustness and high resolution achieved by the proposed method. We also perform a numerical convergence test, a comparison of the solutions of the equations with nonlocal and local fluxes, and a study of the dependence of the numerical solution on the look-ahead distance.

Keywords: Finite volume methods, non-local conservation laws, traffic flow models

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57 Some results on regional optimal control problem for fractional systems with time-varying delays in the state

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Abstract

Mathematically, the system will be represented by a virtual model based on equations and signs, that is to say a model, Fractional systems have been proved, with the development of science and technology, to be one of the most effective tools in modeling many phenomena arising in physics, engineering, and real world problems ,Currently, qualitative properties such as observability, controllability, stability, stabilizability of fractional-order dynamical systems are the issues dealt with by scholars. A fundamental concept widely used in the analysis and design of control systems in modern control theory is Controllability. Adding to that the concept of regional controllability, this would be more interesting especially for researchers who work on applied problems. This concept appears while one is studying or treating a phenomena and maybe he is interested in some regions more than others. This notion has been introduced by El Jai et al. (1995) for parabolic linear systems. We will discuss here the regional controllability of fractional control systems with time-varying delays in the state using the classical Caputo derivative. We first present the considered system and we give some preliminary results and we present some obtained results.

Keywords: regional controllability; fractional-order systems; Caputo derivatives; time-varying delays in the state ;optimal control; minimum energy.

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58 STABILIZATION OF SEMILINEAR SYSTEMS IN BANACH SPACE

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Abstract

In this paper, we aim to study the feedback stabilization of an infinite-dimensional semilinear system evolving in reflexive Banach state space. The concept of bounded control is also investigated in the realistic domain. Sufficient conditions for appropriate feedback control to ensure strong and weak stabilization are given.

Keywords: infinite semi-linear system; duality mapping; feedback controls; strong stabilization; decay estimate; weak stabilization.

59 Non vanishing and Sign changes of Modular Forms ((ID 63))

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Abstract

The goal of this paper is to examine the properties of the (simultaneous) non vanishing and the (simultaneous) sign changes of Fourier coefficients for (two) modular forms called h multiplicative modular forms $f(z) = \sum_{n \geq 1} a(n)q^n \in S_k(\Gamma_0(N))$, satisfying the arithmetic relation

$$a(p^k) = a(p)a(p^{k-1}) - h(p)a(p^{k-2}), \quad a(1) = 1$$

where h is a multiplicative function.

Keywords: Fourier coefficients of cusp forms, Modular forms, Sign change of cusp forms.

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60 On the Non-negative Integer Solutions to Diophantine Equations $F_n - F_m = 7^a$ and $F_n - F_m = 13^a$

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Abstract

In this paper, we study the solutions of the equation $F_n - F_m = p^a$ where p is either 7 or 13 and $n > m \geq 0$, $a \geq 2$. We confirm the conjecture of Erduvan and Keskin by proving that there is no solutions for this Diophantine equation. We will use the lower bounds for linear forms in logarithms (Baker's theory) and a version of the Baker-Davenport reduction method in Diophantine approximation.

Keywords: Diophantine equation, Fibonacci and Lucas numbers, Linear forms in logarithms, continued fraction, Baker's reduction method.

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61 Renormalized solutions for some nonlinear and non-coercive parabolic problem in the anisotropic Sobolev spaces.

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Abstract

This paper is devoted to the study of following nonlinear and noncoercive parabolic problem:

$$\begin{cases} \frac{\partial u}{\partial t} + Au = f - \operatorname{div}(\phi(x, t, u)) & \text{in } \Omega \times (0, T) = Q_T, \\ u(0, x) = u_0(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \times (0, T); \end{cases}$$

in the parabolic anisotropic Sobolev spaces $L^{\vec{p}}(0, T; W_0^{1, \vec{p}}(\Omega))$, where Ω is a bounded open set of \mathbb{R}^N ($N \geq 2$), and $\phi = (\phi_1, \phi_2, \dots, \phi_N)$ is Carateodory operator verifies the nonstandard growth condition and $f \in L^1(Q_T)$. We show the existence of renormalized solutions for this parabolic equation

Keywords: Nonlinear parabolic equations, non-coercive problems, Renormalized solutions. parabolic anisotropic spaces.

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62 Mathematical analysis and numerical simulation of the Ebola Epidemic Disease in The Sense of Conformable Fractional Derivative

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Abstract

The aim of this talk is to present a mathematical model for the response of T-cytotoxic lymphocytes to the Ebola virus using the Herz-tuckwill model with nonlinear conformable fractional order differential equations which derive from real biological data, then study the global stability of the equilibria using an appropriate Lyapunov function and the LaSalle invariance principle. Furthermore, we demonstrate the impact of the non-integer order of the model compared with the integral order. Finally consider a numerical simulation that justifies the biological hypotheses and the theory results.

Keywords: Conformable derivative, Local stability, Exponential stability, Lyapunov function, Fractional epidemic model.

63 Ozone Concentration Forecasting Using LSTM Recurrent Neural Network in Tangier: A Comparative Analysis

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Abstract

This study consists in solving the problem of air quality forecasting in the town of Tangier, by presenting a model based on artificial neural networks. Air pollution problems in Tangier are linked to several pollutants that include the tropospheric ozone (O₃). Which is becoming more worried. Predicting daily concentrations of this pollutant can play a major role in reducing potential risks.[1] This work aims to make a good prediction of daily concentrations of O₃ in the city of Tangier using two types of recurrent neural networks (RNN): the simpleRNN model and the Long Short-Term Memory (LSTM) architecture. In each experiment using these two algorithms, we changed the input data to investigate the relationship between predictors and performance.[2] To evaluate our forecasting model, we graphically represented the predicted and the observed values. The error was quantified using several evaluate indexes: the Mean Square Error (MSE), the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE), and the Index of agreement. The numerical results indicate that both algorithms worked well, but the LSTM model shows that it is more effective at forecasting O₃ concentrations than simple RNN architecture.

Keywords: Air quality forecasting, Artificial Neural Network, Recurrent Neural Networks (RNN), Long Short-Term Memory (LSTM), Ozone (O₃).

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64 Some numerical results of the Brinkman problem using generalised Taylor-Hood elements

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Abstract

This paper presents our numerical results of applying Isogeometric Analysis (IGA) to the velocity-pressure formulation of the steady-state incompressible Brinkman equations. For the approximation of the velocity and pressure fields, LBB-compatible Bspline spaces are used which can be regarded as smooth generalizations of Taylor-Hood pairs of finite element spaces. The lid-driven cavity flow is considered in two dimensions as model problems to investigate the scheme's numerical properties.

Keywords: NURBS, Brinkman problem, convergence-free, isogeometric methods.

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65 New Hybrid Finite Volume Scheme for Air Quality on Vertex-Centred Mesh

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Abstract

The Advection-Diffusion-Reaction equation is commonly used to model air quality problems [3]. Finite Volume schemes have been developed to solve this equation, with some schemes designed for advection or diffusion-dominated problems [2, 4]. However, some existing schemes have limitations in accurately predicting system behavior, particularly in scenarios where there are high pollutant concentrations or complex sources with dominant advection. Due to these limitations, the numerical solutions could become unstable or inaccurate, leading to errors in predicting the behavior of the system. To address these limitations, we introduce a new Hybrid Finite Volume Scheme that can effectively solve the Advection-Diffusion-Reaction equation, even in situations where neither advection nor diffusion is dominant. This scheme is particularly useful for modeling air quality problems, especially when the advection process is dominant over diffusion. We use this new scheme to conduct numerical simulations of air quality problems and illustrate its performance in such applications. This new hybrid Finite Volume scheme has been proposed for vertex-centered meshes. It offers significant improvements in accuracy and computational efficiency, making it a promising approach for air quality problems.

Keywords: Finite Volume methods, Advection-Diffusion-Reaction, Dominant Advection, Air Quality, Advection scheme.

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66 GENERATION OF 2-D UNSTRUCTURED MESH USING A NOVEL ALGORITHM FOR NODE PLACEMENT

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Abstract

The use of meshes has become increasingly important across various industries, including architecture, automotive engineering, and aerospace. Proper node placement is critical for obtaining accurate and efficient numerical simulations in these applications. Node placement methods vary depending on the complexity of the geometry, the desired resolution of the mesh, and the computational resources available. In some cases, simple algorithms, such as regular grids or random placement, may be sufficient. However, more advanced methods, such as optimization-based algorithms or machine learning techniques, may be necessary for complex geometries or high-resolution meshes. Overall, proper node placement is a crucial component of any meshing algorithm, and choosing an appropriate method can significantly impact the quality and efficiency of numerical simulations. In this work, we introduce a novel algorithm for determining the placement of nodes based on a user-defined size function. The creation of well-shaped triangles, similar to Distmesh algorithm[1], can be achieved by connecting the nodes using Delaunay triangulation. However, In the smoothing step, an internode force with an attractive effect is employed. Additionally, this work presents enhancements to Distmesh algorithm, specifically in regards to creating a non-uniform triangular mesh according to a user-defined size function[2,3].

Keywords: Mesh generation, Node placement, Distmesh.

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67 Exploring the Applications of DC Programming in Support Vector Machine Learning

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Abstract

DC programming, a method for breaking down complex optimization problems into simpler ones, and Support Vector Machines (SVMs), a learning algorithm, are both widely used techniques in the field of optimization and machine learning. When combined, these techniques have been proven to be effective in solving various optimization problems, particularly those related to classification and feature selection. This paper highlights the key uses of DC programming in the context of Support Vector Machines (SVMs) for classification and feature selection, and presents the current state-of-the-art results and recent advancements.

Keywords: DC programming, Support Vector Machine (SVM), Feature Selection, Classification.

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68 A NEW CLASS OF WEAK* SEQUENTIALLY COMPACT AND ITS RELATIONSHIP WITH LIMITED OPERATORS

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Abstract

In this paper we introduce and study a new class $W_{s,c}^*(X, Y)$ of operators defined from a Banach space X into a Banach space Y that we call weak* sequentially compact operators. Then, we give some results concerning this class of operators between the Banach lattices. Furthermore, we study the relationship between this class and other classes of operators, some other interesting results are also obtained.

Keywords: Limited operator, Dunford-Pettis operator, Banach lattice, Order continuous norm.

69 Strong Nash implementation in finite and infinite fair allocation problems

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Abstract

Strong implementation is a challenging problem for which there is no theorem similar to that of Maskin (1977/1999) for Nash implementation with simple conditions, which involves a wide range of applications. In this paper, we reexamine the origins of the Maskin theorem (1979). We define the concept of bad options that are non-fixed and are weakly dominated by any socially optimal alternative for $(n - 1)$ players, where at least one player strictly prefers his bad option to those of all other players. We use a weak variant of the no-veto power property that, unlike the standard notion, can be verified for several social choice correspondences (SCCs). The main result establishes that any SCC that has $(n - 1)$ -option and satisfies the properties - weak Pareto optimality, Maskin monotonicity, weak no-veto power, and unanimity - can be implemented in strong equilibria by a strategic game form if there are at least three players. We finally provide applications of our result in finite fair allocation problems for matching markets, and in infinite fair allocation problems under domain restrictions with private values.

Keywords: Implementation, Strong Nash equilibrium, Fair allocation problems.

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70 Central reversible ring property via idempotent elements

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Abstract

In this work, we introduce the concept of central e -reversible rings to extend central reversible rings via idempotent elements. These rings generalize e -reduced rings, e -symmetric rings and e -reversible rings. In this talk, we give some characterizations of central e -reversible rings in various ways and we show that this notion is not left-right symmetric. Moreover, we examine the transfer of central e -reversibility over some various ring extensions.

Keywords: Central e -reversible rings, central reversible rings, e -reversible rings, idempotent elements.

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71 AHP-Method and its applications: Review

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Abstract

Herewe present the analytic hierarchy process (AHP). The Analytic Hierarchy Process (AHP) is a decision-making tool developed by mathematician Thomas Saaty in the 1970s. It is a structured approach that allows individuals to make decisions based on multiple criteria, by breaking down a complex problem into smaller, more manageable parts,we then introduce the principe of this method and some of the central thoeretical. The AHP method has a wide range of applications in various fields, including: Project selection, Resource allocation, Environmental impact assessment, Renewable energy.

Keywords: Analytic Hierarchy Process (AHP); Decision analysis; Multiple criteria analysis.

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72 Network Visualization and Verbalization of Mathematical Problem Solving in Middle School Students: Modeling Problem Solving Networks

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Abstract

Mathematics is a living science and a universal language that allows us to understand reality, and its teaching at all levels develops logical thinking; abstraction capacities and contributes to the acquisition of personal work capacities; self-formation and constructive initiative. This teaching must contribute to the formation of a methodical, inventive and critical human being, gifted in reasoning in a correct and autonomous way, which allows the student to be able to adapt with the cognitive and technological novelties. And since the mastery of the main elements of mathematics is acquired and exercised essentially through problem solving, problem solving has occupied, and still occupies (one of the main competences of the 21st century), a central place in mathematics curricula, either as a means to build new knowledge, to develop general heuristics, to evaluate and validate the acquisition of mathematical knowledge and concepts, or also as an object of teaching in its own right to develop different and essential high level cognitive and metacognitive skills.

Keywords: Mathematics education, problem solving, cognitive processes, modeling, solving networks

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73 Feedback optimal control of a Multi-Objective problem modeling the wastewater treatment.

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Abstract

The process of bio-degradation of organic pollutants by fungi is one of the most important mathematical models from biology. Generally, those models involve optimal open-loop controls, although the control of such problems requires closed loop system. In this work, our aim is to find a feedback control strategy for a wastewater treatment station. We propose a nonlinear multi-objective optimal control problem, describing this process, involving a feedback control depending on the state variable of the pollutant. In the first part, we study the existence of a solution of the dynamic using differential inclusion formulation, then we show that the Pareto front is convex, which leads to identify Pareto optimums. Finally we use the Hamilton Jacobi Bellman equations to show that the value function associated with the scalarized problem admits a unique viscosity solution.

Keywords: Multi-objective optimal control, Feedback control, Differential inclusion, Pareto front, Hamilton Jacobi Bellman, viscosity solution.

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74 Cantor-Bendixson derivatives of the spectrum

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Abstract

The purpose of this presentation is to establish characterizations of Cantor-Bendixson derivatives of the spectrum of bounded linear operators by utilizing the concept of almost invertibility. Specifically, we will demonstrate that zero does not belong to the α -th Cantor-Bendixson derivative of T 's spectrum, denoted by $\text{acc}^\alpha(\sigma(T))$, is equivalent to the impossibility of decomposing T into the direct sum of an invertible operator and an operator with a countable number of elements in its spectrum.

Keywords: Cantor-Bendixson derivative, Almost invertible, Accumulation points, Spectrum

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75 Some Identities of 3-Prime Near-Rings Involving Jordan Ideals and Left multipliers

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Abstract

In this paper we study 3-prime near rings with left multipliers satisfying certain differential identities on Jordan ideals, and we provide examples to show that the assumed restrictions cannot be relaxed.

Keywords: 3-Prime near-rings; Left multipliers; Jordan ideals.

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76 Spectrally starred advertibly complete A - p -normed algebras

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Abstract

If $(E, \|\cdot\|_p)$, $0 \leq p \leq 1$, is an advertible complete A - p -normed algebra each element of which has a convex spectrum, then E modulo its Jacobson radical is isomorphic to \mathbb{C} . In the involutive case, we obtain the same conclusion for an advertible complete A - p -normed algebra under the convexity hypothesis on the spectrum of each normal element only. If the algebra is additionally hermitian, it suffices to assume that the spectrum of each unitary element is convex. The case of algebras with involution anti-morphism are also considered

Keywords: A - p -normed algebra, Q - A - p -normed algebra, Advertible complete algebra, Starred spectrum, Spectrally convex algebra, Jacobson's radical, Involutive algebra.

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77 Subspace- hypercyclicity

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Abstract

We present an M -hypercyclic semigroups on the separable complex Banach spaces X , with M is a subspace of X . The M -hypercyclicity criteria for semigroups are given, as well as the properties of this class of semigroups .

Keywords: Hypercyclicity. Topologically transitive. M -Hypercyclicity. C_0 -semigroup.

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78 Simultaneous continuous functional calculus and applications

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Abstract

We define and study a continuous functional calculus for a commutative family $(a_i)_{i \in I}$ of normal elements of a C^* -algebra. This functional calculus consists in giving a sense to $f(\mathbf{a})$ whenever $\mathbf{a} = (a_i)_{i \in I}$ is a commutative family of normal elements of a C^* -algebra $(A, \|\cdot\|)$ and f is a continuous complex valued function on the simultaneous spectrum $Sp(\mathbf{a})$ of \mathbf{a} . So we show that this functional calculus is unique, continuous and satisfies the spectral mapping theorem. Once this functional calculus is defined and studied, we obtain a particular orthonormal basis on a locally Hilbert space.

Keywords: C^* -algebra, Simultaneous spectrum, Continuous functional calculus, Locally Hilbert space..

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79 Stability and bifurcation of fluid drop subjected to a uniform electric field

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Abstract

We theoretically investigate the stability and bifurcation of a dielectric drop suspended in a second dielectric liquid subject to a uniform electric field. Axisymmetric equilibrium shapes are found by solving simultaneously the Young-Laplace equation at the interface and Laplace equation for the electric field. Analytical solutions are constructed for the governing nonlinear boundary-value problem using domain perturbation method together with a special type of Hermite–Padé approximation.

Keywords: drop deformation, drop breakup, electrohydrodynamic effect, capillary electric number.

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80 Jordan product and fixed points preservers

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Abstract

Let $B(X)$ be the space of all bounded linear operators on an infinite-dimensional complex Banach space X . For $A \in B(X)$, we denote by $F(A)$ the subspace of fixed points of A . In this paper, we study and characterize all surjective maps ϕ on $B(X)$ satisfying

$$F(\phi(T)\phi(A) + \phi(A)\phi(T)) = F(TA + AT)$$

for all $A, T \in B(X)$.

Keywords: Fixed points, Jordan product, Preserver.

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81 A new approach to improve optimizer performance through algorithms diversification for image reconstruction in diffuse optical tomography

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Abstract

In this work, we explore a different way to construct optimizer algorithms for solving the inverse problem of Diffuse Optical Tomography by using diversification of two stochastic gradient-based algorithms, namely NADAM and AMSGrad. We will study the speed of convergence of the proposed newbreed of algorithms, also we will discuss the quality of reconstructed images in both cases of free of noise and noisy measurement data. For analysis and exploration of the potential of the proposed algorithm, we use statistical simulations and analysis approach.

Keywords: Diffuse optical tomography, inverse problem, diversification, random switching algorithm, image reconstruction, statistical simulation.

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82 On the Radial Solutions of a p -Laplace Equation with Hardy Potential

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Abstract

In this paper, we study the asymptotic behavior of radial solutions of the following quasi-linear equation with Hardy potential

$$\Delta_p u + h(|x|) |u|^{p-2} u = 0, \quad x \in \mathbb{R}^N - \{0\},$$

where $2 < p < N$, h is a radial function on $\mathbb{R}^N - \{0\}$ such that $h(|x|) = \gamma |x|^{-p}$, $\gamma > 0$ and $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$ is the p -Laplacian operator. The study strongly depends on the sign of $\gamma - (\sigma/p^*)^p$ where $\sigma = (N - p)/(p - 1)$ and $p^* = p/(p - 1)$.

Keywords: Quasi-Linear Equation, p -Laplacian, Hardy Potential, Radial Solutions, Dynamical System, Characteristic Equation, Asymptotic Behavior.

83 Explicit formulas of the addition law on the elliptic curve over a special nonlocal ring

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Abstract

In this paper, we come to study some properties of elliptic curves over a nonlocal ring $A = \mathbb{F}_q[\varepsilon]; \varepsilon^3 = \varepsilon^2$, where \mathbb{F}_q is the finite field of q elements, and q is a power of a prime integer $p \geq 5$. We discuss, and we define here a new addition law on such elliptic curve over this special ring. More precisely we show that such elliptic curve over A is isomorphic to the product of an elliptic curve over the field F_q and one over the local ring $\frac{\mathbb{F}_{2^d}[X]}{(X^2)}$. This allows us to compute the addition points over these elliptic curves, and to propose such protocol cryptographic.

Keywords: Finite ring, Nonlocal ring, Elliptic curve, Cryptography.

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84 On Galois Extensions of Graded Rings

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Abstract

Throughout this work, $(\Gamma, +)$ is an abelian group and $R = \bigoplus_{\lambda \in \Gamma} R_\lambda$ is a commutative Γ -graded ring. Let B be an R_0 -algebra and $A = R \otimes_{R_0} B$ the graded algebra tensor product of R and B . In this paper, we study Galois theory of some strongly graded algebras. In particular, we show that if G is a finite subgroup of $\text{Aut } B$ and G' its image in $\text{Aut } A$ then B is a G -Galois extension of the ring R_0 if and only if A is a G' -Galois extension of the ring R . As an application we establish some new results on G -Galois extension of commutative CP-graded rings and some commutative algebra over graded field.

Keywords: Quasilinear parabolic equations, non-coercive problems, renormalized solutions.

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85 Stabilization analysis of a mosquito-borne disease mathematical model with discontinuous vaccination and treatment feedback strategies

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Abstract

In this paper, we conduct a stabilization analysis of a mosquito-borne disease mathematical model. The model incorporates a discontinuous feedback mechanism that includes vaccination and treatment. In addition, we formulated it as a system of differential equations that capture the interactions between the human and mosquito populations. To account for the non-linear relationship between mosquito growth and resource consumption, we incorporate general Holling functions in the mosquito subsystem. Our objective is to identify the conditions under which the system will globally asymptotically converge to an almost disease-free equilibrium. The theoretical analysis draws on the Filippov solution, Lyapunov functions, and LaSalle's invariance principle. By incorporating general Holling functions, we improve the accuracy of the model and obtain a more realistic representation of the interactions between the mosquito population and its environment. This study highlights the importance of stabilization analysis when implementing effective interventions against mosquito-borne diseases.

Keywords: Mosquito-Borne disease, Discontinuous Feedback, Filippov solution, Stabilization theory, Lyapunov functions.

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86 Fixed Point Theorem And Application To Integral Equations

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Abstract

The existence of a fixed point for mixed monotone nearly asymptotically nonexpansive mappings is the concern of the present work. We extend and generalize some well-known results concerning nearly asymptotically nonexpansive mappings in a uniformly convex hyperbolic metric space. As application of our obtained results, we study the existence of solutions for the following nonlinear integral equation

$$x(t) = \int_0^1 a(t,s)f(s,x(s))g(x(s)) ds$$

Keywords: Fixed point theorem, Hyperbolic metric space, Monotone nearly asymptotically nonexpansive operator, Integral equation.

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87 Existence result for some coupled nonlinear parabolic systems in Orlicz-Sobolev spaces

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Abstract

Consider the nonlinear parabolic system

$$\begin{cases} \frac{\partial b_i(x, u_i)}{\partial t} - \operatorname{div} \left(\mathcal{A}(x, t, u_i, \nabla u_i) + \Phi_i(x, t, u_i) \right) + f_i(x, u_1, u_2) = 0 & \text{in } Q_T \\ u_i = 0 & \text{on } \Gamma \\ b_i(x, u_i)(t = 0) = b_i(x, u_{i,0}) & \text{in } \Omega, \end{cases}$$

where $i = 1, 2$. In this paper we deal with the renormalized solution for the above system in Orlicz-Sobolev spaces where f_i is a Carathéodory function satisfying some growth assumptions. The main term which contains the space derivatives and a non-coercive lower order term are considered in divergence form satisfying only the original Orlicz growths.

Keywords: Parabolic systems, Orlicz growths, Orlicz-Sobolev spaces, renormalized solutions.

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88 Positive Solutions of an Elliptic Equation Involving a Sign-Changing Potential and a gradient term

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Abstract

The objective of this paper is to investigate the following elliptic singular Laplacian equation

$$\Delta u - |\nabla u|^q + u^p - u^{-\delta} = 0 \text{ in } \mathbb{R}^N,$$

where $N \geq 1, 1 < q < p$ and $\delta > 2$.

Our main contributions consist of establishing the existence of an entire strictly positive solution and analyzing certain properties of its asymptotic behavior, particularly when it exhibits monotonicity.

Keywords: Elliptic equation; Sign-Changing potential, Gradient term; Radial solution; Banach Fixed Point Theorem; Energy Function; Oscillation Methods.

89 ON BOUNDED OPERATORS OF BANACH-JORDAN SYSTEMS

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Abstract

The goal of this paper consists of proving the inclusion of separating subspace of epimorphisms onto a Banach-Jordan system (pair or algebra) in its Jacobson radical. This yields Johnson's Theorem for epimorphisms onto Banach-Jordan systems, namely the uniqueness of their complete norm topology.

Keywords: Banach-Jordan pair, Banach-Jordan algebra, epimorphism, separating subspace, automatic continuity

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90 SOME NEW RESULTS ON (ψ_+, ψ_-) -DERIVATIONS OF BANACH-JORDAN SYSTEM

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Abstract

Generalized Leibniz rule for a ψ -derivation $D = (D_+, D_-)$ on a Jordan pair $V = (V^+, V^-)$ is established. In a second step a local property of the ψ -derivation $D = (D_+, D_-)$ is investigated, namely the quasiniptotency of couples (D_+a, D_-b) of the Banach-Jordan pair $V = (V^+, V^-)$ under the condition $D_+^2a = D_-^2b = 0$. An automatic continuity result is also given.

Keywords: Homomorphism, derivation, ψ -derivation, generalized Leibniz rule, Jordan algebra, Jordan pair, Banach-Jordan pair

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91 The Aluthge and mean transform on certain class of bounded linear operators

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Abstract

Let us consider $B(H)$ the full algebra of bounded linear operator on a Hilbert space H . For an operator $T \in B(H)$, we consider its polar decomposition $T = V|T|$, where $V \in B(H)$ is the associate partial isometry with the kernel condition, and $|T|$ is the square root of the operator T^*T . The Aluthge transform $\Delta(T)$ and the Mean transform \widehat{T} of the operator T are respectively defined by :

$$\Delta(T) = |T|^{\frac{1}{2}}V|T|^{\frac{1}{2}} \quad \text{and} \quad \widehat{T} = \frac{V|T| + |T|V}{2}.$$

In this talk, we discuss the image and preimage by Aluthge and mean transform of some class of operators such as positive, normal, unitary, hyponormal, and co-hyponormal operators.

Keywords: Bounded Operators, Polar decomposition, Aluthge transform, Mean transform, Class of bounded operators.

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92 Application of J.F.Colombeau theory

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Abstract

In this paper we proved some importance proprieties of Colombeau algebra, we proved the existence and uniqueness of solution of transport equation with variable speed and initial data in the Colombeau algebra G . We proved the association of the generalized solution with the classical solution.

Keywords: Colombeau algebra , Generalized solution , Association , Transport equation .

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93 Numerical methods for an inverse shape design problem with Robin boundary condition

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Abstract

The purpose of this study is to perform the mathematical analysis of a free boundary problem with robin boundary condition, besides to the development and comparison of different methods for solving this inverse problem. First, the shape optimization problem is established, then we prove the existence of an optimal solution. The first conditions of optimality are computed, after that we compute the shape gradient and hessian. The finite element method is proposed for the discretization of the state and adjoint equations. Two deterministic approaches are considered, the conjugate gradient and the quasi newton methods, also we perform their convergence analysis. Two heuristic approaches are proposed, the differential evolution and genetic algorithms. Besides we propose a hybrid minimization technique, which is a combination of differential evolution with the quasi newton method. Several numerical examples are established to prove the validity of proposed approaches, then a comparison of the obtained results is realized to show the differences and deduce which approach is robust over the rest.

Keywords: Conjugate Gradient, Differential Evolution, Free Boundary problem, Hybrid methods, Genetic Algorithms, Quasi Newton, Shape Optimization.

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94 Combining Non-Classical Method and Finite Element Method for Efficient Numerical Solution of Time-Fractional Diffusion Equation

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Abstract

In this paper, we consider the numerical resolution of a time-fractional diffusion equation using the finite element method in space and an improved quadrature method in time. The proposed method is based on the work of Carolin Birk and Chongmin Song[2], which uses a non-classical kernel function to approximate the Caputo fractional derivative. In our approach, we extend this idea by incorporating the finite element method in space to discretize the partial differential equation. In addition, we use an improved quadrature method to approximate the time fractional derivative in the temporal domain. Our numerical experiments demonstrate that our proposed method is more efficient than the standard direct method. Specifically, our new scheme has the same order of convergence as the standard direct scheme, but with a much smaller step size in time and a geometrically smaller computational complexity in space. As a result, our proposed method significantly reduces the CPU time compared to the standard direct scheme. We provide numerical experiments to illustrate the effectiveness of our proposed method.

Keywords: Anisotropic Sobolev spaces, non-coercive quasilinear elliptic equation, unilateral problems, entropy solutions.

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95 On the Positive Radial Solutions of p-Laplacien Problem on an Unbounded Exterior Domain

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Abstract

The present paper establishes the existence of positive radial solutions to the p-Laplacien equation: $\Delta_p u - u + u^q = 0$, $p > 2$ and $q > 1$, on an unbounded exterior domain $\Omega_e \subset \mathbb{R}^N$, with zero initial datum.

First, we provided a result on the existence of positive radial solutions using a shooting method and an associated energy function. Second we derived some important results on the asymptotic behavior of global solutions at infinity.

Keywords: p-Laplacian operator, Radial solution, Existence of solutions, Shooting method, Energy function, Asymptotic behaviour.

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96 Improved statistical and machine learning methods for images database classification

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Abstract

The image classification is a classical problem of image processing, computer vision and machine learning fields. In this paper we study the image classification using deep learning. We use ANN architecture with convolutional neural networks for this purpose. Four test images are selected from the ImageNet database for the classification purpose. We cropped the images for various portion areas and conducted experiments. The results show the effectiveness of deep learning based image classification using ANN.

Keywords: Machine Learning, images classification, optimization, statistical methods, Convolutional neural networks.

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97 An existence result for two-dimensional parabolic integro-differential equations involving CEV model

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Abstract

In this paper, we present an existence result of weak solutions for some parabolic equations involving the so-called CEV model with jumps

Keywords: Option pricing, CEV model, model with jumps, integro-differential degenerate parabolic equations.

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98 Pricing American options with irregular Payoff via reflected BSDEs

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Abstract

We formulate the value function of American options with irregular Payoff in term of the unique solution of an extended reflected backward stochastic differential equations (RBSDEs in short) where the barrier process is assumed to be regular (with regulated trajectories) .

Keywords: American options, irregular Payoff, Reflected BSDEs.

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99 ESTIMATION OF MISSING ELEMENTS IN PAIRWISE COMPARISON MATRIX USING GENETIC ALGORITHM

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Abstract

This paper concerns the incomplete pairwise comparison matrix produced from judgments of experts in the Analytic Hierarchy Process (AHP). In such situation, we have to estimate the missing elements with suitable values by ensuring the consistency of the pairwise matrix. Metaheuristics are powerful tool to solve optimization problems. Then, improved genetic algorithm (GA) is proposed to recover the missing judgments until a satisfactory level of consistency is reached. Experiment results are performed with incomplete pairwise matrices with different sizes showing the effectiveness of our algorithm.

Keywords: Genetic Algorithm, Pairwise Matrix, Analytic Hierarchy Process, Decision theory

100 Generalized Solution of Burger Equation

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Abstract

In this paper, the existence and uniqueness of the generalized solution of the Burger equation is studied with initial conditions are distributions (elements of Colombeau algebra). Then we study the association concept with the classical solution.

Keywords: Burger equation, Generalized solution, Distributions, Colombeau algebra, Association.

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101 Generalized Solution of Burger Equation

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Abstract

In this paper, the existence and uniqueness of the generalized solution of the Burger equation is studied with initial conditions are distributions (elements of Colombeau algebra). Then we study the association concept with the classical solution.

References

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102 Periodic solutions in shifts δt for impulsive neutral dynamic equations with infinite delay on time scales

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Abstract

Let Π be a periodic time scale in shift δ_{\pm} . We use the Krasnoselskii's fixed point theorem to show that the impulsive neutral dynamic equations with infinite delay

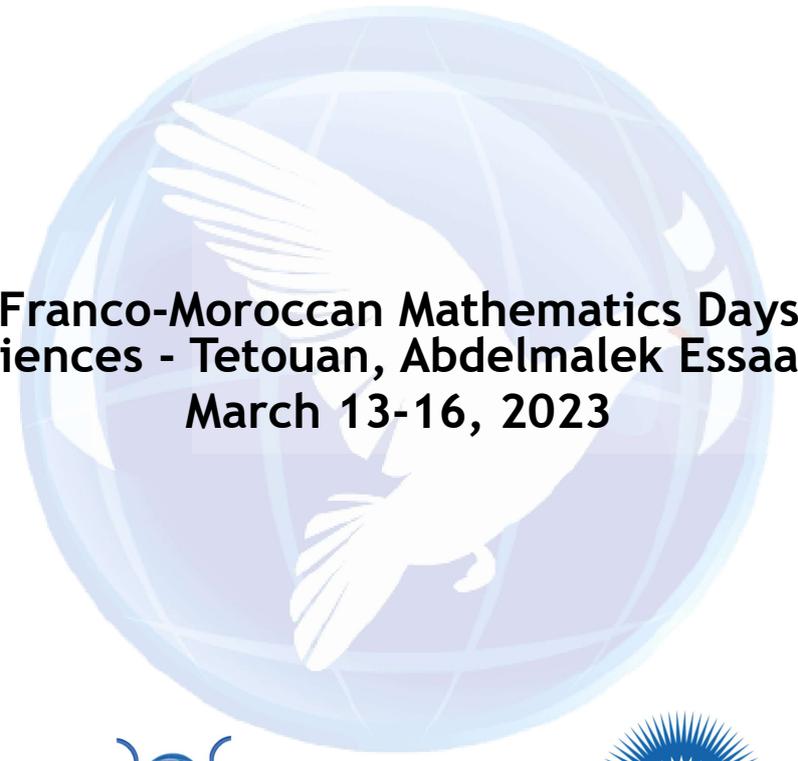
$$\begin{cases} x^{\Delta}(t) = -A(t)x^{\sigma}(t) + f^{\Delta}(t, x(t-h(t))) + \int_{-\infty}^t B(t,s)g(x(s))\Delta s, & t_k \neq t \in \Pi, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k)), & k \in \mathbb{Z}^+. \end{cases}$$

have a periodic solution in shift δ_{\pm} . Under a slightly stringent conditions we show that the periodic solution in shifts δ_{\pm} is unique using the contraction mapping principle.

Keywords: Impulse, delta dynamic equation, periodic solution, shift operator, Krasnoselskii's fixed point theorem, time scale.



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